Walking on Data Words

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Introduction

Origins and applications of data languages

- **Databases**: to model specifications and queries on XML documents that contain both symbols from a finite alphabet and data values from a potentially infinite alphabet
- **Verification**: to model properties of programs parametrized by variables ranging over an infinite domain
- **Formal languages**: to understand how the classical theory of regular languages over finite alphabets can be extended to languages over infinite alphabets.

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What precisely is a data language?

A **data language** is a set of finite words over an infinite alphabet $\Sigma \times \mathbb{D}$ of pairs of letters and data values.

To make life easier, one usually enforces the following restriction:

Languages are invariant under permuations of data values

$$(\text{e.g.} \quad \binom{a}{1}\binom{b}{2}\binom{a}{2}\binom{b}{1}\binom{a}{3} \in L \quad \text{iff} \quad \binom{a}{5}\binom{b}{3}\binom{a}{3}\binom{b}{5}\binom{a}{7} \in L \quad)$$

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An example of data language

$$\mathrm{L} \;=\; \left\{ w \in \mathrm{D}^* \;\middle|\; w \textit{ contains at least } 2 \textit{ distinct values} \right\}$$

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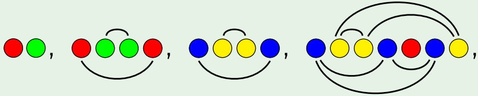

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- **deterministic** / **non-deterministic** / **alternating**
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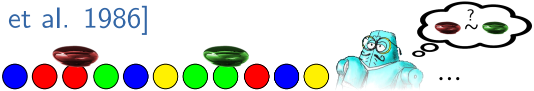
- **Pebbles** [Chang et al. 1986]



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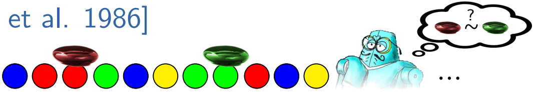
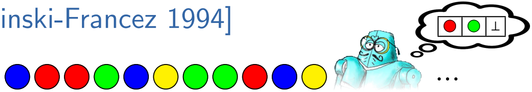


- **Registers** [Kaminski-Francez 1994]



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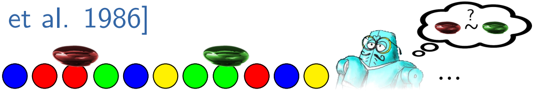

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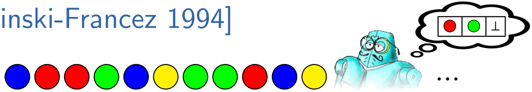
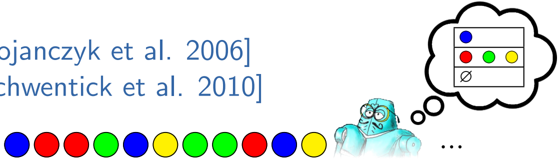


- **Hash tables** [Bojanczyk et al. 2006]
  [Schwentick et al. 2010]



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Unlike classical automata:

- different models have often different expressive power
- difficult to get equivalence between automata and logics
- there is a tradeoff between
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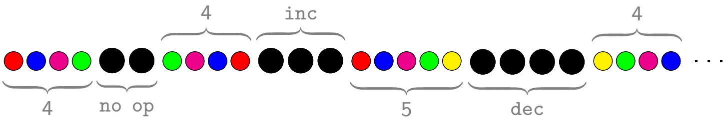
Examples for automata with registers

- One-way deterministic $\Rightarrow$ quite weak (no reversals)
- One-way non-deterministic $\Rightarrow$ no complementation
- One-way alternating $\Rightarrow$ emptiness undecidable
- Two-way deterministic $\Rightarrow$ emptiness undecidable

Introduction

☞ A word on undecidability of emptiness...

...it often arises from the possibility of encoding counter machines:



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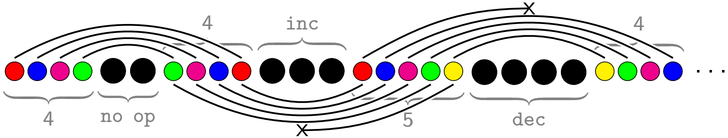

In some sense, data words are problematic because they can be seen as graphs with potentially unbounded grid minors.

Outline

In this talk:

Data Walking Automata

Closures and other properties

Decision problems

Data Walking Automata

We think of a data word as a special graph:

- vertices are positions in the word:

- 4 types of edges:

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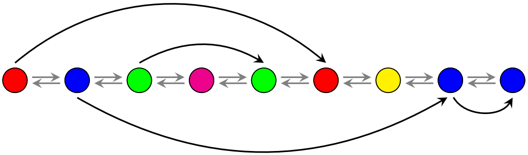

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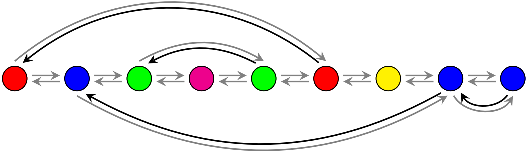

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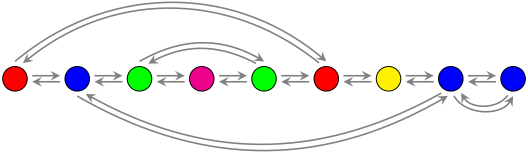

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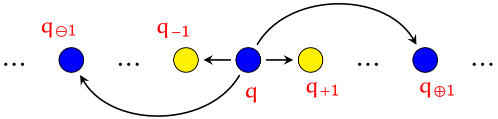
☞ Vertices are implicitly labeled with **local types** specifying whether successors/precedessors exist and whether they coincide.

Data Walking Automata

2 natural notions of automata on data words seen as graphs:

1. **Tiling Automata** [Thomas 1997]
   a.k.a. **Data Automata** [Bojanczyk et al. 2006]
   or **Class Memory Automata** [Schwentick et al. 2010]

   runs = labelings satisfying constraints on neighborhoods



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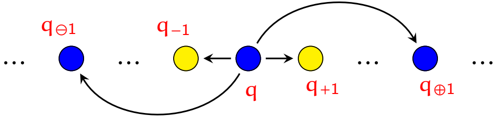
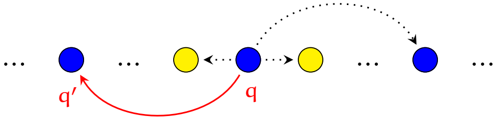


2. **Walking Automata** [Aho, Ullman 1971]

   runs = deterministic / non-deterministic traversals



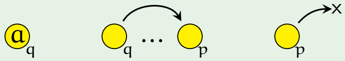
Data Walking Automata

Example 1

$\mathrm{L} = \{$ *words where all* a *are followed by* b *in the <u>same</u> class* $\}$

- A **Tiling Automaton** marks the longest b-suffix in each class:

  uses states p, q, avoids tiles



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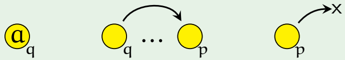
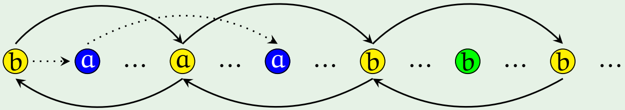


- A **Walking Automaton** scans each class verifying $\{a, b\}^*\{b\}^+$:



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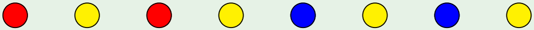
$$\mathtt{L} = \left\{ \textit{words where all } \mathtt{a} \textit{ are followed by } \mathtt{b} \textit{ in a } \underline{\textit{different}} \textit{ class} \right\}$$

To accept a data word, e.g.



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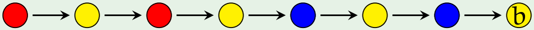

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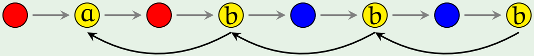

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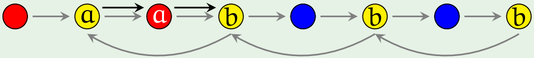

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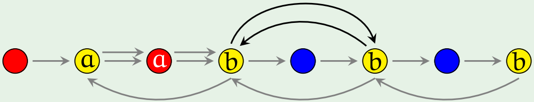

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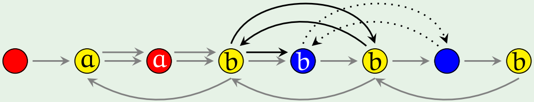

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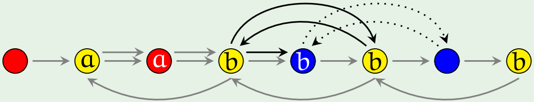

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5. accept iff reached position is not the last.

Closures and other properties

Proposition

Non-deterministic Data Walking Automata are effectively closed under **union** and **intersection**.

Deterministic Data Walking Automata are effectively closed under **union**, **intersection**, and **complementation**.

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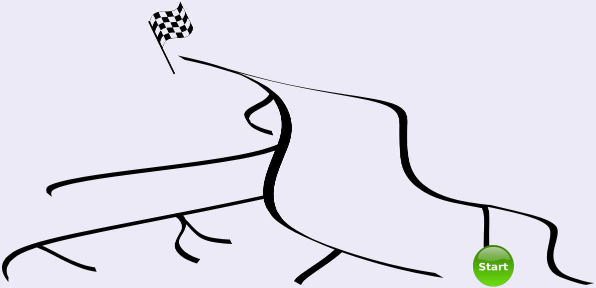

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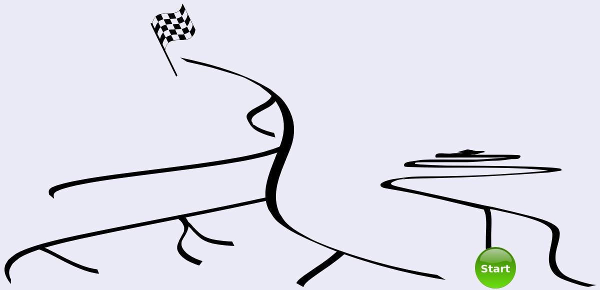

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Data Walking	$\subsetneq$	Data Walking	$\not\subseteq$	Automata
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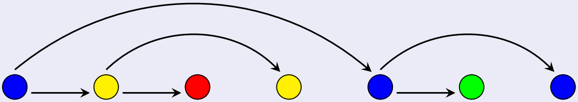
Reduce to analogous results [Colcombet and Bojanczyk 2006,2008] for Tree Walking Automata, by **encoding trees** with data words:



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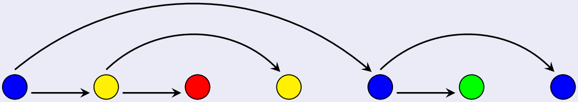

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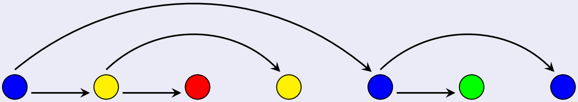

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Corollary

Tiling Automata recognize **projections** of languages recognized by Deterministic Data Walking Automata.

Corollary

Emptiness of Deterministic Data Walking Automata is as hard as emptiness of Tiling Automata and **Petri Nets reachability**.

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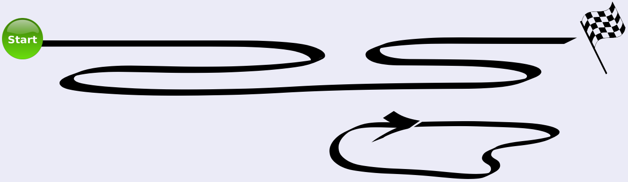

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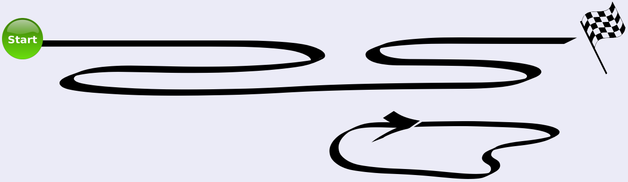

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Emptiness is decidable for Non-det. Data Walking Automata.

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Consider any set of vertices (positions + state) that

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☞ The automaton rejects the input data word iff the final configuration is not in the above set

☞ The above set is a special form of tiling

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Universality is decidable for Non-det. Data Walking Automata.

Decision problems

☞ Tiling Automata are closed under unions and intersections.

Corollary

Containment and, more generally, any boolean combination of Non-deterministic Data Walking Automata is decidable.

The general picture